

M-Theory on Complex Spacetime

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Abstract

In this paper we will analyse ABJM theory in $N = 1$ superspace formalism on complex spacetime. We will then analyse the BRST and anti-BRST symmetries for this theory. We will show that the sum of gauge fixing and ghost terms for this theory can be expressed as a combination of the total BRST or the total anti-BRST variations.

1 Introduction

Gauge symmetry plays an important role in nature [2]-[10]. It is also important in understanding multiple M2 branes [11]-[23], and multiple M5 branes [24]-[25]. Gauge symmetry can be analysed either via the Wheeler-DeWitt approach [55]-[56] or the BRST approach [32]-[42]. Recently interest in complex spacetime has been generated due to certain developments in the string theory [43]-[52]. Complex spacetime has been studied as a model for non-symmetric gravity formalism [47, 48]. Nonanticommutative field theories and nonanticommutative quantum gravity have been formulated on this complex spacetime [53]-[54].

The action for M -theory at low energies is a superconformal action with manifest $N = 8$ supersymmetry. This action was discovered by Bagger and Lambert and was based on gauge symmetries generated by a Lie 3-algebra [55]-[59]. However, only one example of such a 3-algebra is known and so far the rank of the gauge group has not been increased. So, a $U(N)_k \times U(N)_{-k}$ superconformal Chern-Simons-matter theory with level k and $-k$ with arbitrary rank and $N = 6$ supersymmetry was also constructed [61]. This theory called the ABJM theory. It is also thought to describe the action of M -theory as it reduced to the Bagger-Lambert theory for the only known example of Lie 3-algebra. Its supersymmetry can also get enhanced to $N = 8$ supersymmetry [60]. Furthermore, a $SO(8)$ R -symmetry at Chern-Simons levels $k = 1, 2$ also exists for this model.

BRST and anti-BRST symmetries for the ABJM theory has been studied [12, 18]. This symmetry for the in deformed superspace. In this paper we will study the BRST and anti-BRST symmetries for the ABJM theory on deformed complex spacetime. It will be demonstrated that on complex spacetime, just like in the the deformed superspace, the sum of the gauge fixing and ghost terms can be expressed as a total BRST and anti-BRST variation.

2 Deformation of ABJM Theory

In this section we will deform the superspace of ABJM theory on complex spacetime. We will now impose the following non-anticommutativity relation [54]

$$\{x^\mu, x^\nu\} = 2x^\mu x^\nu + i\tau^{\mu\nu}, \quad (1)$$

where $\tau^{\mu\nu} = \tau^{\nu\mu}$. This leads to the following star product

$$X(x, \theta) \diamond Y(x, \theta) = \exp \left[\frac{i}{2} \tau^{\mu\nu} \partial_\mu^1 \partial_\nu^2 \right] X(x_1, \theta) Y(x_2, \theta)_{x_1=x_2=x}. \quad (2)$$

It is also useful to define the following bracket

$$2[X, Z]_\diamond = X \diamond Z \pm Z \diamond X, \quad (3)$$

where the relative sign is negative unless both the fields are fermionic. The super-derivative D_a is defined by

$$D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b, \quad (4)$$

Now we define the spinor superfieds Γ_a and $\tilde{\Gamma}_a$ as

$$\begin{aligned} \Gamma_a &= \chi_a + B\theta_a + \frac{1}{2}(\gamma^\mu)_a A_\mu + i\theta^2 \left[\lambda_a - \frac{1}{2}(\gamma^\mu \partial_\mu \chi)_a \right], \\ \tilde{\Gamma}_a &= \tilde{\chi}_a + \tilde{B}\theta_a + \frac{1}{2}(\gamma^\mu)_a \tilde{A}_\mu + i\theta^2 \left[\tilde{\lambda}_a - \frac{1}{2}(\gamma^\mu \partial_\mu \tilde{\chi})_a \right]. \end{aligned} \quad (5)$$

We also define scalar superfieds X^I and $X^{I\dagger}$, with

$$\begin{aligned} \nabla_{(X)}^a \diamond X^I &= D_a X^I + i\Gamma_a \diamond X^I - i\tilde{\Gamma}_a \diamond X^I, \\ \nabla_{(X)}^a \diamond X^{I\dagger} &= D_a X^{I\dagger} - i\Gamma_a \diamond X^{I\dagger} + i\tilde{\Gamma}_a \diamond X^{I\dagger}, \end{aligned} \quad (6)$$

Now we the ABJM theory given by

$$\mathcal{L}_c = \mathcal{L}_M + \mathcal{L}_{CS} - \tilde{\mathcal{L}}_{CS}, \quad (7)$$

where

$$\begin{aligned} \mathcal{L}_{CS} &= \frac{k}{2\pi} \int d^2\theta \operatorname{Tr}[\Gamma^a \diamond \omega_a], \\ \tilde{\mathcal{L}}_{CS} &= \frac{k}{2\pi} \int d^2\theta \operatorname{Tr}[\tilde{\Gamma}^a \diamond \tilde{\omega}_a], \end{aligned} \quad (8)$$

and

$$\begin{aligned} \omega_a &= \frac{1}{2} D^b D_a \Gamma_b - i[\Gamma^b, D_b \Gamma_a]_\diamond - \frac{1}{3}[\Gamma^b, [\Gamma_b, \Gamma_a]_\diamond]_\diamond \\ \tilde{\omega}_a &= \frac{1}{2} D^b D_a \tilde{\Gamma}_b - i[\tilde{\Gamma}^b, D_b \tilde{\Gamma}_a]_\diamond - \frac{1}{3}[\tilde{\Gamma}^b, [\tilde{\Gamma}_b, \tilde{\Gamma}_a]_\diamond]_\diamond. \end{aligned} \quad (9)$$

Furthermore,

$$\mathcal{L}_M = \frac{1}{4} \int d^2\theta \operatorname{Tr} \left[\nabla_{(X)}^a \diamond X^{I\dagger} \diamond \nabla_{a(X)} \diamond X_I + \frac{4\pi}{k} \mathcal{V}[X^{I\dagger}, X^I]_\diamond \right], \quad (10)$$

where $\mathcal{V}[X^{I\dagger}, X^I]_\diamond$ is the potential term with the product of all fields replaced by the star product.

3 BRST and anti-BRST Symmetry

Some of the degree's of freedom in the Lagrangian are not physical. This is because of the following gauge transformations,

$$\begin{aligned}\delta \Gamma_a &= \nabla_a \diamond \Lambda, & \delta \tilde{\Gamma}_a &= \tilde{\nabla}_a \diamond \tilde{\Lambda}, \\ \delta X^I &= i(\Lambda \diamond X^I - X^I \diamond \tilde{\Lambda}), & \delta X^{I\dagger} &= i(\tilde{\Lambda} \diamond X^{I\dagger} - X^{I\dagger} \diamond \Lambda),\end{aligned}\quad (11)$$

here

$$\nabla_a = D_a - i\Gamma_a, \quad \tilde{\nabla}_a = D_a - i\tilde{\Gamma}_a. \quad (12)$$

So, we add the following gauge fixing term to the original Lagrangian density,

$$\mathcal{L}_{gf} = \int d^2\theta \operatorname{Tr} \left[b \diamond (D^a \Gamma_a) + \frac{\alpha}{2} b \diamond b - i\tilde{b} \diamond (D^a \tilde{\Gamma}_a) + \frac{\alpha}{2} \tilde{b} \diamond \tilde{b} \right]_{|}. \quad (13)$$

We also add the following ghost term

$$\mathcal{L}_{gh} = \int d^2\theta \operatorname{Tr} [\bar{c} \diamond D^a \nabla_a \diamond c - \tilde{c} \diamond D^a \tilde{\nabla}_a \diamond \tilde{c}]_{|}. \quad (14)$$

The total Lagrangian density obtained this way is invariant under the following BRST transformations,

$$\begin{aligned}s \Gamma_a &= \nabla_a \diamond c, & s \tilde{\Gamma}_a &= \tilde{\nabla}_a \diamond \tilde{c}, \\ s c &= -[c, c]_{\diamond}, & s \tilde{c} &= -\tilde{b} - 2[\tilde{c}, \tilde{c}]_{\diamond}, \\ s \bar{c} &= b, & s \tilde{c} &= -[\tilde{c}, \tilde{c}]_{\diamond}, \\ s b &= 0, & s \tilde{b} &= -[\tilde{b}, \tilde{c}]_{\diamond}, \\ s X^I &= i(c \diamond X^I - X^I \diamond \tilde{c}), & s X^{I\dagger} &= i(\tilde{c} \diamond X^{I\dagger} - X^{I\dagger} \diamond c),\end{aligned}\quad (15)$$

and the following BRST transformations,

$$\begin{aligned}\bar{s} \Gamma_a &= \nabla_a \diamond \bar{c}, & \bar{s} \tilde{\Gamma}_a &= \tilde{\nabla}_a \diamond \tilde{\bar{c}}, \\ \bar{s} c &= -b - 2[\bar{c}, c]_{\diamond}, & [\bar{s}, \tilde{c}]_{\diamond} &= \tilde{b}, \\ \bar{s} \bar{c} &= -[\bar{c}, \bar{c}]_{\diamond}, & \bar{s} \tilde{c} &= -[\tilde{c}, \tilde{c}]_{\diamond}, \\ \bar{s} b &= -[b, c]_{\diamond}, & \bar{s} \tilde{b} &= 0, \\ \bar{s} X^I &= i(\bar{c} \diamond X^I - X^I \diamond \tilde{\bar{c}}), & \bar{s} X^{I\dagger} &= i(\tilde{\bar{c}} \diamond X^{I\dagger} - X^{I\dagger} \diamond \tilde{c}).\end{aligned}\quad (16)$$

Both these sets of transformations are nilpotent.

$$[s, s]_{\diamond} = [\bar{s}, \bar{s}]_{\diamond} = 0. \quad (17)$$

We can now write

$$\begin{aligned}\mathcal{L}_{gf} + \mathcal{L}_{gh} &= - \int d^2\theta \bar{s} \operatorname{Tr} \left[c \diamond \left(D^a \Gamma_a - \frac{i\alpha}{2} b \right) - \tilde{c} \diamond \left(D^a \tilde{\Gamma}_a - \frac{i\alpha}{2} \tilde{b} \right) \right]_{|} \\ &= \int d^2\theta s \operatorname{Tr} \left[\bar{c} \diamond \left(D^a \Gamma_a - \frac{\alpha}{2} b \right) - \tilde{\bar{c}} \diamond \left(D^a \tilde{\Gamma}_a - \frac{\alpha}{2} \tilde{b} \right) \right]_{|}. \quad (18)\end{aligned}$$

Thus, the sum of gauge fixing and ghost terms can be expressed either as a total BRST variation or as a total anti-BRST variation.

4 Conclusion

In this paper we studied the ABJM theory in $N = 1$ superspace formalism. We have then studied the deformation of this theory on deformed complex spacetime. We analysed the BRST and the anti-BRST symmetries for this theory. It was shown that the sum of the ghost and gauge fixing terms can be written as a total BRST or a total anti-BRST variation. It will be interesting to perform this analyses with non-linear BRST and anti-BRST transformations. It is known that for ABJM theory on a deformed superspace, in Landau and Non-linear gauges, the sum of the gauge fixing and ghost terms can be expressed as a combination of both BRST and anti-BRST transformations. It will be interesting to derive a similar result for the ABJM theory on complex spacetime.

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